Corrigé du devoir



I. 1°/. $D^2 a_n$.

$D^k a_n =$

2°/. a) $D f_{n,k} = k f_{n+1,k-1}$.

$I_0 f_{n,k} = \frac{1}{k+1} \cdot f_{n,k+1}$

b). $D g_{n,r} = r^n .(r-1) = (r-1).g_{n,r}$.

$I_0 g_{n,r} = \frac{1}{r-1}(r^{n+1} - 1) = \frac{1}{r-1}(g_{n+1,r} - 1)$.

$r = e^{i\theta}$. $I_0 g_{n,r} = e^{in\theta/2} . \frac{\sin(n+1)\theta/2}{\sin \theta/2}$

$I_0 c_{n,\theta} = \frac{\cos(n\theta/2) \sin(n+1)\theta/2}{\sin \theta/2} \Rightarrow I_0 c_{n,\theta} - 1/2 = \frac{\sin(n+1/2)\theta}{2 \sin \theta/2}$

II

1°/. a) $D a_n = 0 \Leftrightarrow a_n = c$.

$D a_n = f_{n,k} . \Leftrightarrow a_n = \frac{1}{k+1} . f_{n,k+1} + c$

$D a_n = g_{n,r} \Leftrightarrow a_n = \frac{1}{r-1} g_{n,r} + c$.

$D a_n = b_n . \Leftrightarrow a_n = I_0 b_{n-1} + c$.

b) $a_{n+1} = 2a_n^2 - 1$ : $a_n = \cos(2^n \theta)$.

2°/. a).

b) i). $D a_n = 0$ : $a_n = \frac{\beta}{1-\alpha}$.

ii) $a_n = \alpha^n . a_0$. ; iii) $a_n - \frac{\beta}{1-\alpha} = \alpha^n(a_0 - \frac{\beta}{1-\alpha})$

3°/. a) $a_n = a_1 . S_n$.

b). $D b_n = \frac{a_{n+1}}{S_{n+1}} - \frac{a_n}{S_n} = \frac{q(n)}{S_{n+1}} \Rightarrow b_n = \sum_{k=1}^{n-1} \frac{q(k)}{S_{k+1}} + a_1$

d'où $a_n$.

c) i) $a_{n+1} = n a_n/(n+1) + n \Rightarrow a_n = a_1/n + \frac{1}{3}(n^2 - 1)$

ii). $a_1 = -1/6$. $a_n = -\frac{1}{(n+1)(n+2)}$