I . Nombres complexes : ①

1°. Soit $r \in \mathbb{C}$ la raison $\Rightarrow z_2 = z_1 . r \quad z_3 = z_1 . r^2$

$\Rightarrow z_1(1+r+r^2) = 0 \iff r^3 = 1 \quad r \neq 1$

3 $\quad z_1 z_2 z_3 = z_2^3 = 8e^{i\pi/3} \Rightarrow z_2 = 2e^{i\pi/9}.\alpha$ avec $\alpha^3 = 1$

$\Rightarrow (z_1, z_2, z_3) = 2e^{i\pi/9}(\bar{r}\alpha, \alpha, r\alpha)$. $r^3 = \alpha^3 = 1$.

ou : $\boxed{\{z_1, z_2, z_3\} = \{2e^{i\pi/9}, 2je^{i\pi/9}, 2j^2e^{i\pi/9}\}}$

2°. $1 + i\sqrt{3} = 2e^{i\pi/3}$

$1 + i(\sqrt{2}-1) = \sqrt{2}.\left[e^{-i\pi/4} + e^{i\pi/2}\right] = 2\sqrt{2}.e^{i\pi/8}(e^{3i\pi/8} + e^{-3i\pi/8})$

$= 2\sqrt{2}\, e^{i\pi/8}.\cos(3\pi/8)$.

$\cos\frac{3\pi}{8} = \frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{\sqrt{2}}} \quad \left(\cos^2 3\pi/8 = \frac{1}{2}(1+\cos 3\pi/4)\right)$.

3 $\quad \Rightarrow u = \sqrt{1-\frac{1}{\sqrt{2}}} \cdot \frac{\sqrt{2}\, 2e^{i\pi/3}}{2\sqrt{2}\, e^{i\pi/8}\sqrt{1-\frac{1}{\sqrt{2}}}} = e^{i\pi/3 - i\pi/8}$

$\boxed{|u| = 1 \qquad \operatorname{Arg} u = \frac{5\pi}{24}}$ 2,5

$u^{1984} = e^{4i\pi/3} = -\frac{1}{2}(1 + i\sqrt{3})$. 0,5

II. Etude de fonctions :

1°. a) $D_f = \mathbb{R} - \{-2\}$ $\quad x > 0 \; f'(x) = 1 + \frac{1}{x+2} > 0$

$x < 0 : f'(x) = -1 + \frac{1}{x+2} = -\frac{(x+1)}{x+2}$

x	$-\infty$		x_2		-2		x_1		-1		0		$+\infty$
f'		$-$		$-$	‖	$+$		$+$	0	$-$	$-\frac{1}{2}$ ‖ $\frac{3}{2}$	$+$	
f	$+\infty$	↘	0	↘ $-\infty$	‖	$-\infty$ ↗	0	↗	1	↘	$\ln 2$	↗	$+\infty$

2

$x_1 \simeq -1,84140566$

$x_2 \simeq -2,12002824$