III.

1°). $D^2 a_n = 0$: $b_n = \alpha$. $\Rightarrow$ $a_n = n\alpha + \beta$.

$b_n = \ln a_n$: $\Rightarrow$ $b_n = n\alpha + \beta$ $\Rightarrow$ $a_n = C.a^n$. $C > 0, a > 0$.

$a_0 < 0 \Rightarrow a_n < 0$: $\Rightarrow$ $a_n = C a^n$: $C < 0$, a de signe qq.

2°). a). $\lambda = \cancel{\gamma}$, $\mu = 2\cancel{\gamma} + \beta$, $\nu = \alpha + \beta + \gamma$.

b) $D^2 g_{n,r} = (r-1)^2 g_{n,r}$: $\cancel{\gamma}.(r-1)^2 + (2\cancel{\gamma} + \beta)(r-1) + \alpha + \beta + \gamma = 0$

$$\Leftrightarrow \cancel{\gamma} r^2 + \beta r + \cancel{\alpha} = 0.$$

c). {sol.} e.v. (r_1^n) et (r_2^n) base.

$r_1 \neq r_2$. $\Rightarrow a_n = \lambda r_1^n + \mu r_2^n$.

$r_1 = r_2 = r$: $a_n = \lambda r^n + \mu n r^n$.

d). i). On pose : $a_n = \lambda r_1^n + \mu r_2^n$ $\Rightarrow$ $\lambda = \frac{-r_1}{\sqrt{5}}$, $\mu = \frac{r_2}{\sqrt{5}}$.

d'où : $a_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$.

ii). $a_n = \frac{1}{3}\left[4^n - 1\right]$.

iii). $a_n = \cos n\theta$ (directement).

3°). a). $W_{n+1} = q(n).W_n$ $\Rightarrow$ $W_n = Q_n(a_0 b_1 - a_1 b_0)$

b) $a_{n+2} D x_{n+1} = a_n . q(n) D x_n \Leftrightarrow D x_n = Q(n) . \frac{a_0 a_1}{a_{n+1} a_n}$

et $x_n = a_0 a_1 \sum_{k=0}^{n-1} \frac{Q(k)}{a_{k+1} a_k} + x_0$.

c) . i). l'ensemble des sol. est donné par :

$a_n = \lambda + \mu . \sum_{k=1}^{n-1} (-1)^{k-1} \frac{k.k!}{2^{k-1}}$. $n \geq 2$. et $a_0 = a_1$.

ii). $a_0 = 0$. et

$$a_n = \lambda . n + \mu n \sum_{k=1}^{n-1} \frac{1}{(k-1)!\, k}$$

faux

$$a_n = \lambda n + \mu n \sum_{i=1}^{n-1} \frac{1}{(i+1)!\,(i-1)!}$$

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