$x \to -\infty$: $\operatorname{sh}(\lambda x) \sim -\frac{1}{2} e^{-\lambda x}$ $\quad 1+\operatorname{ch} x \sim \frac{1}{2} e^{-x}$ $\Rightarrow \boxed{f_\lambda(x) \sim -e^{(1-\lambda)x}}$

d'où : $\lambda < 2$ $\quad f_\lambda(x) \to -\infty$: branche parabolique d'axe Oy

$\lambda = 2$ $\quad f_\lambda(x) \to -1$: asymptote : $y = -1$

$\lambda > 2$ $\quad f_\lambda(x) \to 0$: asymptote : Ox.

c) On trouve : $f'_\lambda(x) = \frac{e^x \operatorname{sh}(\lambda x)}{1+\operatorname{ch} x}\left[1 + \lambda \coth \lambda x - \frac{\operatorname{sh} x}{1+\operatorname{ch} x}\right] = \frac{e^x \operatorname{sh} \lambda x}{1+\operatorname{ch} x}\left[g_\lambda(x)\right]$.

$\frac{f'_\lambda(x)}{x}$ est du signe de $g_\lambda(x)$:

Or : $g'_\lambda(x) = -\operatorname{th}^2\frac{x}{2} - \frac{\lambda^2}{\operatorname{sh}^2 \lambda x} < 0 \Rightarrow$

x	$-\infty$		0
$g'_\lambda(x)$		$-$	
$g_\lambda(x)$	$-\lambda+2$	$\searrow$	$-\infty$

(si $x > 0$, $f'_\lambda(x) > 0$).

$\lambda \geq 2$.

x	$-\infty$		0		$+\infty$
$f'_\lambda(x)$		$+$	$\lambda/2$	$+$	
$f_\lambda(x)$	$-\infty$ (-1 si $\lambda = 2$)	$\nearrow$	0	$\nearrow$	$+\infty$

$0 < \lambda < 2$

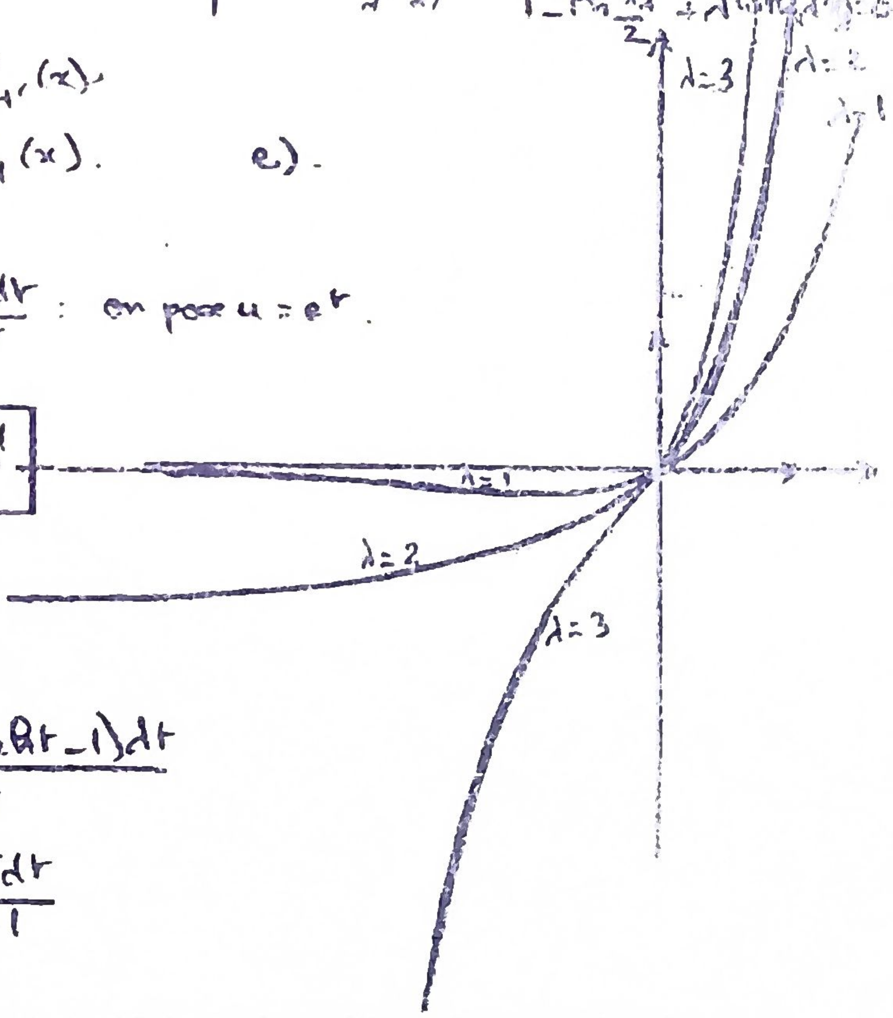
x	$-\infty$		x_λ		0		$+\infty$
$f'_\lambda(x)$		$-$	0	$+$	$\lambda/2$	$+$	
$f_\lambda(x)$	0	$\searrow$	$f_\lambda(x_\lambda)$	$\nearrow$	0	$\nearrow$	$+\infty$

x_λ vérifie : $1 - \operatorname{th}\frac{x_\lambda}{2} + \lambda \coth(\lambda x_\lambda) = 0$

d) $\lambda < \lambda'$: $x > 0$: $\operatorname{sh} \lambda x < \operatorname{sh} \lambda' x$: $f_\lambda(x) < f_{\lambda'}(x)$.

$x < 0$: $\operatorname{sh} \lambda x > \operatorname{sh} \lambda' x$: $f_{\lambda'}(x) < f_\lambda(x)$.

e).



2°) a) $F_1(x) = \int_0^x \frac{e^t \operatorname{sh} t}{1+\operatorname{ch} t}\,dt = \int_0^x \frac{(e^t - e^{-t})e^t\,dt}{2 + e^t + e^{-t}}$: on pose $u = e^t$.

$F_1(x) = \int_1^{e^x} \frac{u^2-1}{(u+1)^2}\,du = \boxed{e^x - 1 - 2\ln\frac{e^x+1}{2}}$

d'où :

$\lim_{x\to+\infty} F_1(x) = +\infty$ et $\lim_{x\to-\infty} F_1(x) = 2\ln 2 - 1$

b) $F_2(x) = \int_0^x \frac{e^t \operatorname{sh} 2t\,dt}{1+\operatorname{ch} t} = \int_0^x \frac{e^t\, 2\operatorname{sh} t\,(1+\operatorname{ch} t - 1)\,dt}{1+\operatorname{ch} t}$

$= \int_0^x e^t . 2\operatorname{sh} t\,dt - 2\int_0^x \frac{(e^{2t}-1)e^t\,dt}{e^{2t}+2e^t+1}$

$= \left[\frac{1}{2}e^{2t} - t\right]_0^x - 2\int_0^x \frac{(e^t-1)e^t}{e^t+1}\,dt$

$= \boxed{\frac{1}{2}e^{2x} - 2e^x - x + 4\ln(e^x+1) + 3/2 - 4\ln 2}$

d'où : $\lim_{x\to+\infty} F_2(x) = +\infty$ $\left(F_2(x) \sim \frac{1}{2}e^{2x}\right)$ et $\lim_{x\to-\infty} F_2(x) = +\infty$ $(F_2(x) \sim -x)$

c) D'après le 1°) b). $f_\lambda(x) \underset{+\infty}{\sim} e^{\lambda x}$ donc, $\exists x_0$, $\forall x > x_0$: $f_\lambda(x) > \frac{1}{2}e^{\lambda x}$

D'où : $F_\lambda(x) = \underbrace{\int_0^{x_0} f_\lambda(t)\,dt}_{A_\lambda} + \int_{x_0}^x f_\lambda(t)\,dt > A_\lambda + \int_{x_0}^x \frac{1}{2}e^{\lambda t}\,dt = A_\lambda + \frac{1}{2\lambda}\left(e^{\lambda x} - e^{\lambda x_0}\right)$

donc, $\lim_{x\to+\infty} F_\lambda(x) = +\infty$.