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Corrigé du devoir surveillé :

I. Calcul de I :

$$\frac{64x}{(x-1)(x^2+2x+5)^2} = \frac{1}{x-1} + \frac{-8x+40}{(x^2+2x+5)^2} + \frac{-x-3}{x^2+2x+5} \leftarrow 4$$

(10)

et en posant $t = \frac{x+1}{2} \Rightarrow \boxed{I = \ln|x-1| + \frac{6x+10}{x^2+2x+5} + 2\,\text{Arctg}\frac{x+1}{2} - \frac{1}{2}\ln(x^2+2x+5) + C}$

Calcul de J : 2 méthodes : $y = t(1-x) \Rightarrow \boxed{J = \frac{2}{\sqrt3}\,\text{Arctg}\,\sqrt3\sqrt{\frac{x+5}{1-x}} + C} \leftarrow 16$

$J = 2\int \frac{dt}{3t^2+1}$.

ou : $x+2 = 3t \Rightarrow J = \int \frac{dt}{(t+2)\sqrt{1-t^2}}$ $\Rightarrow \boxed{J = \frac{2}{\sqrt3}\,\text{Arctg}\frac{x+8-2\sqrt{-x^2-4x+5}}{(x+2)\sqrt3} + C}$

II. Calcul d'une primitive de A :

$I = \int \frac{dx}{\sin x\sqrt{2\cos x-1}}$ $\quad t = \sqrt{2\cos x - 1}$ $\Rightarrow I = 4\int\frac{dt}{(t^2-1)(t^2+3)} = -\frac{1}{2}\ln\frac{1+\sqrt{2\cos x-1}}{1-\sqrt{2\cos x-1}} - \frac{1}{\sqrt3}\text{Arctg}\sqrt{\frac{2\cos x-1}{3}} + C$ (8)

$$\Rightarrow \boxed{A = \frac12\ln\left|\frac{1+\sqrt{\sqrt3-1}}{1-\sqrt{\sqrt3-1}}\right| + \frac{1}{\sqrt3}\,\text{Arctg}\sqrt{\frac{\sqrt3-1}{\sqrt3}}} \simeq 1{,}54158 \leftarrow 4$$

Calcul de B : on pose $u = \pi - x \Rightarrow B = \frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\sin^2x}dx = \frac{\pi}{2}\int_{-1}^{+1}\frac{du}{2-u^2}$ $(u=\cos x)$ (4)

$$B = \frac{\pi}{2\sqrt2}\int_{-\frac{1}{\sqrt2}}^{\frac{1}{\sqrt2}}\frac{dv}{1-v^2} = \frac{\pi}{2\sqrt2}\ln\frac{\sqrt2+1}{\sqrt2-1} = \frac{\pi\sqrt2}{2}\ln(\sqrt2+1) \leftarrow 4 \quad B \simeq 1{,}957919836$$

III.

1°/. $A(x) = \text{Arctg}\frac{1-\cos x}{\sin x} + \text{Arctg}(\cot g\, x)$ $\quad x \in]0,\pi[\leftarrow 5$

$\lim\limits_{x\to0} A(x) = \frac{\pi}{2}$. $\leftarrow 2$

Expression simplifiée de $A(x)$: $\frac{1-\cos x}{\sin x} = \text{tg}\frac{x}{2}$; $\cot g\,x = \text{tg}(\frac{\pi}{2}-x)$

$\Rightarrow \boxed{A(x) = \frac{\pi-x}{2}}$ $\quad$ rep. graphique ! $\leftarrow 5$

2°/. $\frac{\sin x}{1-2t\cos x+t^2} = \sin x + t\sin 2x + \dots + t^{n-1}\sin nx + \left[\frac{t^n\sin(n+1)x - t^{n+1}\sin nx}{1-2t\cos x+t^2}\right] \leftarrow 6$

$\Rightarrow A(x) = X_n + R_n$ où : $R_n = \int_0^1\frac{t^n\sin(n+1)x - t^{n+1}\sin nx}{1-2t\cos x+t^2}dt$

Il suffit de montrer que $\lim\limits_{n\to+\infty} R_n = 0$

Or : $|t^n\sin(n+1)x - t^{n+1}\sin nx| \leq 2t^n$ $\quad (|\sin ax| \leq 1$ et $t^{n+1} \leq t^n)$. $\leftarrow 6$

$\Rightarrow |R_n| \leq \int_0^1\frac{2t^n\,dt}{1-2t\cos x+t^2}$ $\quad$ étudions : $y(t) = t^2 - 2t\cos x + 1$ sur $[0,1]$

$\Rightarrow y(t) \geq \sin^2 x$.

$\Rightarrow |R_n| \leq \frac{2}{\sin^2x}\cdot\int_0^1 t^n dt = \frac{2}{(n+1)\sin^2x}$ $\Rightarrow \lim\limits_{n\to+\infty}R_n = 0$ $\quad (x \in]0,\pi[)$

3°/. Il suffit de prendre $x = \frac{\pi}{2}$.

$\sin\left(\frac{2k+1}{2}\pi\right) = (-1)^k$
$\sin(k\pi) = 0$
$\Rightarrow u_n = X_{2n+1}(\pi/2) \Rightarrow \lim\limits_{n\to+\infty}u_n = A(\frac{\pi}{2}) = \frac{\pi}{4}$. $\leftarrow 6$

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