Corrigé du devoir surveillé.

I. (19 pts)

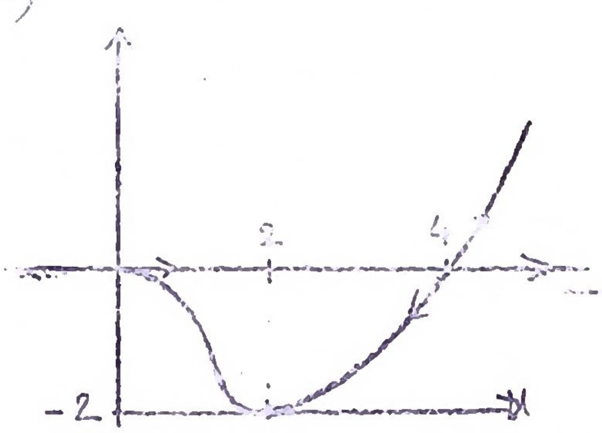
1°/. $e^{a+x} = e^a e^x = e^a\left(1 + x + \frac{x^2}{2} + \dots + \frac{x^n}{n!} + o(x^n)\right)$ (3 pts).

2°/. $D_f = \mathbb{R}^*$; $f'(x) = \frac{e^{\frac{x-2}{x}}}{x^2}(x^2+2x-8)$ (5 pts)

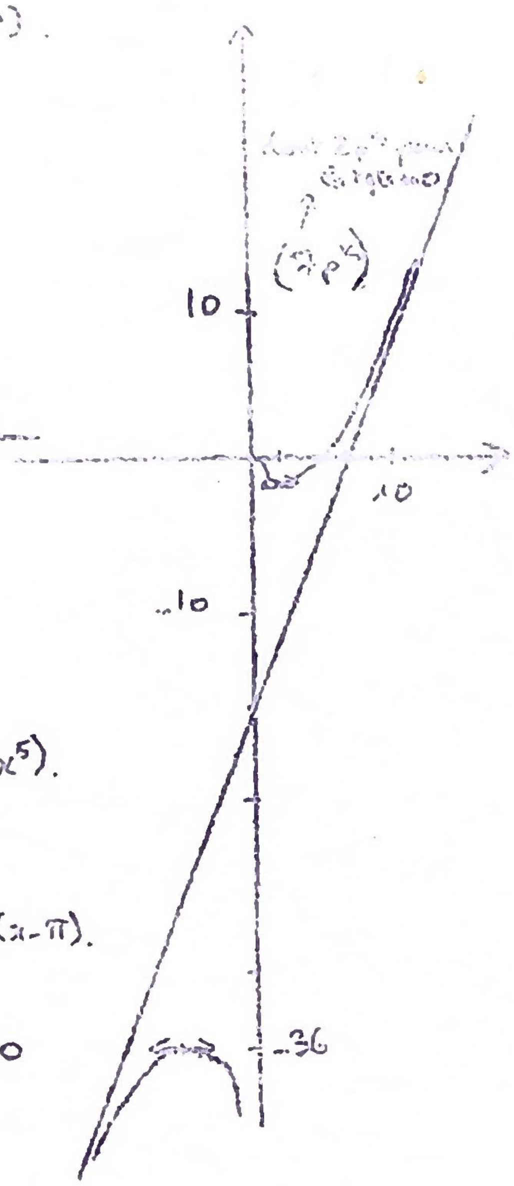
Tableau de variations:

x	$-\infty$		-4		0		2		$+\infty$
$f'(x)$		$+$	0	$-$	$\|\|$	$-$	0	$+$	
$f(x)$	$-\infty$	$\nearrow$	$-8e^{3/2}$	$\searrow$	$-\infty \,\|\|\, 0$	$\searrow$	-2	$\nearrow$	$+\infty$

D.L.G. ds $V_{+\infty}$:
$f(x) = e\left(x - 6 + \frac{10}{x} + o\left(\frac{1}{x}\right)\right)$. (4 pts)



au vois. de 0.



II. (61 pts). A (24 pts)

1°/. i) $f(x) = \dfrac{x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^6)}{\sqrt{1 + 2x^2 - \frac{x^4}{6} + o(x^5)}} = x - \frac{7}{6}x^3 + \frac{211}{120}x^5 + o(x^5)$. (4 pts).

ii) En utilisant la formule de T. Y.
à l'ordre 1 : $f(x) = f(\pi) + (x-\pi)f'(\pi) + o(x-\pi)$ $\Rightarrow f(x) = -\frac{1}{3}(x-\pi) + o(x-\pi)$. (2 pts).
(ou un changement de variable pour se ramener en 0).

2°/. $f(x) - \sin x = f(x)\left[1 - \sqrt{5 - 4\cos x}\right] = f(x) \cdot \dfrac{-4(1-\cos x)}{1 + \sqrt{5 - 4\cos x}}$; $f(x) \geq 0$, $-4(1-\cos x) \leq 0$, $1 + \sqrt{5-4\cos x} \geq 0$

$\Rightarrow f(x) - \sin x \leq 0$.

$f(x) - x = f(x) - \sin x + (\sin x - x)$: or : $\sin x \leq x$ (pour $x \geq 0$) $\Rightarrow f(x) - x \leq 0$.

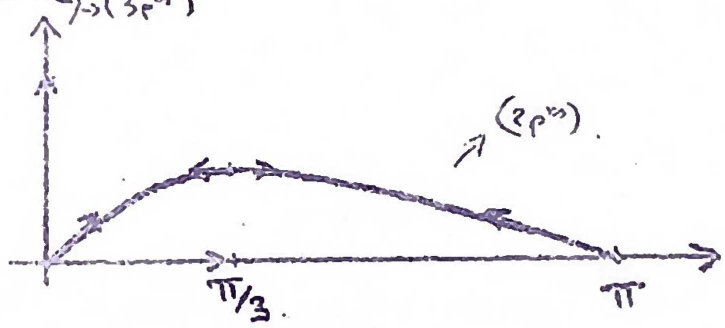
$f(x) = x \iff f(x) = \sin x$ et $\sin x = x \iff x = 0$. (5 pts).

3°/. $f'(x) = \dfrac{2 - \cos x}{(5 - 4\cos x)^{3/2}} \cdot (2\cos x - 1)$ · du signe de $2\cos x - 1$. (3 pts)

Tableau de variations: (2 pts)

x	0		$\pi/3$		π
$f'(x)$	1	$+$	0	$-$	$-1/3$
$f(x)$	0	$\nearrow$	$1/2$	$\searrow$	0

(7 pts).



4°/. $\dfrac{1}{f(x)} = \dfrac{1}{x} \cdot \dfrac{1}{1 - \frac{7x^2}{6} + \frac{211}{120}x^4 + o(x^4)} = \frac{1}{x}\left(1 + \frac{7x^2}{6} - \frac{143}{360}x^4 + o(x^5)\right) \Rightarrow r(x) = \frac{7}{6}x - \frac{143}{360}x^3 + o(x^3)$. (3 pts).

$s(x) = r(x) \cdot \left(\dfrac{1}{f(x)} + \dfrac{1}{x}\right) = \dfrac{7}{3} + \dfrac{17}{30}x^2 + o(x^2)$ (3 pts)

(6 pts).

B (37 pts).

1°/. g est définie ssi : $-1 \leq \dfrac{4 - 5\cos x}{5 - 4\cos x} \leq 1 \iff \begin{cases} 1 + \cos x \geq 0 \\ 9(1 - \cos x) \geq 0 \end{cases}$ (car $5 - 4\cos x \geq 0$). OK! (2 pts)

$\left.\begin{array}{l}\cos(\mathrm{Arccos}\, u) = u \\ \sin(\mathrm{Arccos}\, u) = \sqrt{1 - u^2}\end{array}\right\} \Rightarrow \cos g(x) = \dfrac{4 - 5\cos x}{5 - 4\cos x}$ (1 pts) et $\sin g(x) = \dfrac{3\sin x}{5 - 4\cos x}$ (2 pts).

(5 pts)